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Gauge-Yukawa Unification and the Top-Bottom Hierarchy †

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Abstract

The consequences of Gauge-Yukawa Unification (GYU) in supersymmetric unified models on low energy physics are analyzed. We find that the observed top-bottom mass hierarchy can be explained by supersymmetric GYU and different models can be experimentally distinguished if the top quark mass lies slightly below its infrared value.

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In Grand Unified Theories (GUTs), the gauge interactions of the standard model (SM) are unified at a certain energy scale $M_{\rm GUT}$, and this unification scheme has given specific testable predictions [1]. The accurate measurements of the gauge couplings at LEP in fact suggest that the minimal $N = 1$ supersymmetric $SU(5)$ GUT [2] is promising when comparing its theoretical values with the experiments.

By a Gauge-Yukawa Unification (GYU) we mean a functional relationship among the gauge and Yukawa couplings, which can be derived from some principle. In superstring and composite models for instance, such relations could be derived in principle. In the GYU scheme [3, 4, 5], which is based on the principle of finiteness and reduction of couplings, one can write down relations among the gauge and Yukawa couplings in a more concrete fashion. (Note that the gauge and Yukawa sectors in GUTs are usually not related.) These principles are formulated within the framework of perturbatively renormalizable field theory, and one can reduce the number of independent couplings without introducing necessarily a symmetry, thereby improving the calculability and predictive power of a given theory[1].

The consequence of GYU is that in the lowest order in perturbation theory the gauge and Yukawa couplings above $M_{\rm GUT}$ are related in the form

$$g_i \;=\; \kappa_i \, g_{\rm GUT} \;,\; i = 1, 2, 3, e, \cdots, \tau, b, t \;, \tag{1}$$

where g_i $(i = 1, \cdots, t)$ stand for the gauge and Yukawa couplings, $g_{\rm GUT}$ is the unified coupling, and we have neglected the Cabibbo-Kobayashi-Maskawa mixing of the quarks. So, Eq. (1) exhibits a boundary condition on the the renormalization group evolution for the effective theory below $M_{\rm GUT}$, which we assume to be the minimal supersymmetric standard model (MSSM). It has been recently found [3, 5] that various supersymmetric GUTs with GYU in the third generation can predict the bottom and top quark masses that are consistent with the experimental data. This

[1] In ref. [6], interesting renormalization group (RG) invariant relations among the soft supersymmetry breaking parameters has been found. These relations are obtained on the close analogy of our approach presented here.

means that the top-bottom hierarchy could be explained in these models, exactly in the same way as the hierarchy of the gauge couplings of the SM can be explained if one assumes the existence of a unifying gauge symmetry at $M_{\rm GUT}$ [1].

It has been also observed [3, 5] that there exists a relatively wide range of k's which gives the top-bottom hierarchy of the right order. Of course, the existence of this range is partially related to the infrared behavior of the Yukawa couplings [8]. Therefore, a systematic investigation on the nature of GYU is indispensable to see whether a GYU can make experimentally distinguishable predictions on the top and bottom masses, or whether the top-bottom hierarchy results mainly from the infrared behavior of the Yukawa couplings. With more precise measurements of the top and bottom masses, we will be able to conclude which case is indeed realized.

We have performed an exhaustive analysis on this problem at the two-loop level [9], and here we would like to present only a few representative results to provide an idea of our complete analysis. We have assumed that below $M_{\rm GUT}$ the evolution of couplings is governed by the MSSM and that there exists a unique threshold $M_{\rm SUSY}$ for all superpartners of the MSSM so that below $M_{\rm SUSY}$ the SM is the correct effective theory, where we include only the logarithmic and two-loop corrections for the RG evolution of couplings [2]. We have neglected all the threshold effects. Note that with a GYU boundary condition alone the value of $\tan\beta$ can not be determined; usually, it is determined in the Higgs sector, which however strongly depends on the supersymmetry braking terms. In our analysis we avoid this by using the tau mass, along with M_Z, $\alpha_{\rm em}^{-1}(M_Z)$ and $\sin^2\theta_{\rm W}(M_Z)$, as the input.

In Table, we present the predictions for the $SU(5)$ type GYU (i.e. $k_1 = k_2 = k_3 = 1, k_b = k_\tau$) with $M_{\rm SUSY} = 500$ GeV and $k_b = \sqrt{(6/5)} \simeq 1.10$ fixed, where we vary k_t from 0.4 to 2.0. The Finite Unified Theory based on $SU(5)$ [3] corresponds

[2] When the threshold effects are appropriately taken into account, the minimal supersymmetric model based on $SU(5)$ predicts a value for the QCD coupling at M_Z that is slightly larger than the experimental one [7]. Similar problem could exist here too. But we ignore this problem, because we do not consider any specific models.

to $k_t = \sqrt{(8/5)} \simeq 1.26$.

Table

k	$\alpha_3(M_Z)$	$m_b(M_b)$ [GeV]	M_t [GeV]	$\tan\beta$
0.6	0.118	4.66	150.2	53.6
0.8	0.120	4.65	166.6	53.5
1.0	0.121	4.62	176.4	53.6
$\sqrt{(8/5)}$	0.122	4.57	184.0	53.8
1.4	0.122	4.55	186.5	53.9
1.6	0.122	4.51	189.3	54.1
2.0	0.123	4.44	192.9	54.5

$M_{t,b}$ are the pole masses while $m_b(M_b)$ is the running bottom quark mass at its pole mass. The values for $m_b(M_b)$ in the table should not be taken very seriously because due to large values of $\tan\beta$ there will be a relatively large correction coming from the superpartner contribution which is not included above. We find that, because of the infrared behavior of the Yukawa couplings [8], above $k_t \simeq 1.6$ (in the case at hand) the value of M_t becomes no longer sensitive against the change of k_t. So, in general, if the experimental value for M_t is close to the infrared value, it is unclear whether the top-bottom hierarchy results from the infrared behavior or from a GYU. (For the example above, the infrared value is about 195 GeV.) On contrary, if M_t is smaller than that value, the GYU might be realistic. Detailed studies on this problem will be published elsewhere [9].

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